Derivation of Ishbash Formula for stone size on sloping apron

Shield's formula

Critical tractive shear for horizontal bed

$$\tau_c = K_s(\gamma_s - \gamma_w)d$$

Critical tractive shear for sloping side reduces by a fraction K_i

$$\tau_{\theta c} = K_i K_s(\gamma_s - \gamma_w)d$$

Where,

$$K_i = \frac{\tau_{\theta c}}{\tau_c} = \sqrt{1 - \frac{sin^2\theta}{sin^2\phi}}$$

Shear stress exerted by flowing water

$$\tau_o = \gamma_w R S_f$$

for wide rivers, $R \simeq y, \quad S_f = \frac{n^2 v^2}{y^{\frac{4}{3}}}$

Substituting $\quad \tau_o = \gamma_w y \frac{n^2 v^2}{y^{\frac{4}{3}}} = \gamma_w \frac{n^2 v^2}{y^{\frac{1}{3}}}$

For incipient motion,

$$\tau_{\theta c} = \tau_o \longrightarrow \gamma_w \frac{n^2 v^2}{y^{\frac{1}{3}}} = K_i K_s (\gamma_s - \gamma_w) d \longrightarrow v^2 = K_i K_s \left(\frac{\gamma_s}{\gamma_w} - 1\right) \frac{d y^{\frac{1}{3}}}{n^2}$$

$$v = \sqrt{K_i K_s} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}} \left(\frac{d^{\frac{1}{2}} y^{\frac{1}{6}}}{n}\right)$$

As per Strickler's formula, $n = K_n d^{\frac{1}{6}}$; where $K_n = \dfrac{\left(\frac{y}{d}\right)^{\frac{1}{6}}}{21.9 log\left(12.2\frac{y}{d}\right)}$

$$v = \sqrt{K_i K_s} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}} \left(\frac{d^{\frac{1}{2}} y^{\frac{1}{6}}}{K_n d^{\frac{1}{6}}}\right) \longrightarrow v = \sqrt{K_i K_s} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}} \left(\frac{d^{\frac{1}{2}}}{K_n} \left(\frac{y}{d}\right)^{\frac{1}{6}}\right)$$

$$v = \left(\frac{\sqrt{K_i K_S}}{K_n} \left(\frac{y}{d}\right)^{\frac{1}{6}} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}}\right) d^{\frac{1}{2}} \longrightarrow v = K_{ds} d^{\frac{1}{2}}$$

Where, $K_{ds} = \dfrac{\sqrt{K_i K_S}}{K_n} \left(\frac{y}{d}\right)^{\frac{1}{6}} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}} = \dfrac{\sqrt{K_i K_S}}{K_n} \left(\frac{y}{d}\right)^{\frac{1}{6}} \left(\frac{\gamma_s}{\gamma_w} - 1\right)^{\frac{1}{2}}$

$$d = \frac{v^2}{K_{ds}^2}$$

Weight of stone W $(Ishbash\ formula)$

$$W = 1000\frac{\pi}{6}S_s d^3 = 1000\frac{\pi}{6}S_s\left(\frac{v^2}{K_{ds}^2}\right)^3$$

$$W = 1000\frac{\pi}{6}S_s\frac{v^6}{\frac{(K_iK_S)^3}{K_n^6}\frac{y}{d}(S_s-1)^3} \rightarrow W = \left(\frac{1000\pi}{6\frac{K_S^3 y}{K_n^6 d}}\right)\frac{S_s v^6}{K_i^3(S_s-1)^3}$$

$$W = \left(\frac{1000\left(\frac{22}{7}\right)}{6\frac{(0.05945)^3}{(0.0461)^6}1.03}\right)\frac{S_s v^6}{K_i^3(S_s-1)^3} \rightarrow \boldsymbol{W = (0.02323)\frac{S_s v^6}{K_i^3(S_s-1)^3}}$$